FAR BEYOND

MAT122

Fundamental Theorem of Calculus (FTC)

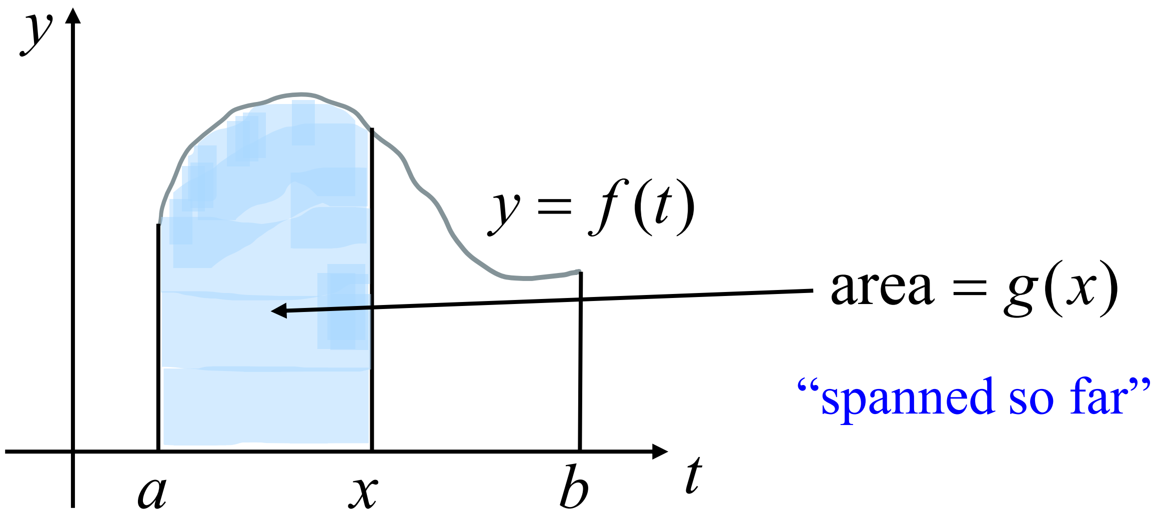
Stony Brook University

Fundamental Theorem of Calculus

The **Fundamental Theorem of Calculus** establishes a connection between <u>differential calculus</u> and <u>integral calculus</u>. They are inverse processes.

FTC Part 1: $\int_a^x f(t)dt = g(x)$ where f is continuous on $[a, b]$.

If f is a <u>positive</u> function then $g(x)$ can be interpreted as the area under the graph of f from a to x where x can vary from a to b.

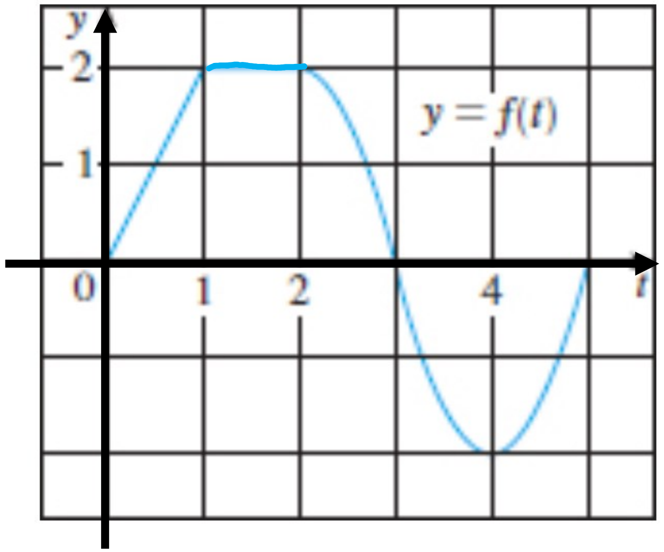


i.e., $g(x)$ is an antiderivative of f :

$$g'(x) = f(x)$$

for $a < x < b$

FTC Part 1 - Example



graph is represented by $g(x) = \int_0^x f(t)\,dt$

Find value of $g(0) = \int_0^0 f(t)\,dt = \boxed{0}$

Find value of $g(1) = \int_0^1 f(t)\,dt = \boxed{1}$

Find value of $g(2) = \int_0^2 f(t)\,dt = \boxed{3}$

Find value of $g(3) = \int_0^3 f(t)\,dt$

$= \boxed{4.3}$

Find value of $g(4)$

$= \boxed{3}$

Find value of $g(5)$

$= \boxed{1.7}$

Differentiating an Integral

FTC Part 1:

$$\int_a^x f(t)dt = g(x)$$

where f is continuous

ex. Find the derivative of $g(x) = \int_0^x \sqrt{1+t^2}\ dt$

$$\boxed{g'(x) = \sqrt{1+x^2}}$$

ex. Find $S'(x)$ when $S(x) = \int_0^x 7t^3\ dt$

$$\boxed{\therefore S'(x) = 7x^3}$$

Differentiating an Integral with Chain Rule

When upper bound is not a simple variable "x", chain rule is necessary.

ex. Find $\dfrac{d}{dx}\displaystyle\int_1^{x^4} \ln t \; dt$

$$= \boxed{4x^3 \ln x^4}$$

Chain Rule without u-substitution

When upper bound is not a simple variable "x", chain rule is necessary.

ex. Find $\dfrac{d}{dx}\displaystyle\int_{7}^{x^2}\left(3+t^2\right)\,dt$

$$=\boxed{2x\left(3+x^4\right)}$$

FTC with Negation

ex. Find the derivative of $\int_x^{\pi} \sqrt{1+e^t}\ dt$

negate

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

switch a and b

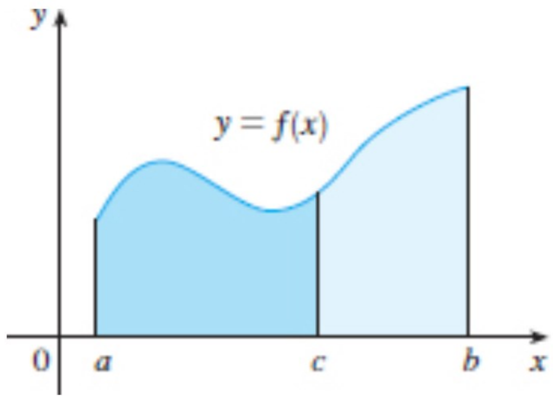
$$= \boxed{-\sqrt{1+e^x}}$$

FTC with Variables in Both Bounds

both bounds are variables

ex. Find the derivative of $\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\, du$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$



$$= \boxed{\frac{3\left(9x^2-1\right)}{9x^2+1} - \frac{2\left(4x^2-1\right)}{4x^2+1}}$$

FTC - Do

Do: Find the derivative of $\int_{x^3}^{1} \sqrt{7t^2 - 3t + 6}\, dt$

Do: Find the derivative of $\int_{4x}^{9x} \ln t \; dt$